Solution of DPP # 8

TARGET : JEE (ADVANCED) 2015
COURSE : VIJAY & VIJETA (ADR & ADP)

1. The energy per photon is

$E_\lambda = \dfrac{hc}{\lambda} = \dfrac{6.6\times10^{-34} m^2kg/s \times 3\times10^8 m/s}{3.3\times10^{-7} m} = 6 \times 10^{-19}$ J.

The time to discharge the plate is given by total number of electrons divided by the rate of photons:

$t = Q/e \times \dfrac{E_\lambda}{P} = \dfrac{6\times10^{-19} J}{1.6\times10^{-19}\times 60 J/s} = 0.0625$ s

2. (i) Nuclear fission results in fragments whose neutron/proton ratio is higher than the required value (N/P ratios is greater for heavier nuclei). To reduce the N/P ratio these fragments undergo β^- decays in which a neutron is converted into a proton.
(ii) Some of the energy generated in α–emission goes into nuclear excitation. The excited nucleus returns to ground state by γ–emission.
(iii) In carbon–carbon cycle, ^{12}C nucleus acts just as a catalyst. The net result is fusion of four protons into a helium nucleus.

3. (C) Velocity at highest point = u sinθ

$\therefore \quad \lambda_D = \dfrac{h}{mu\sin\theta}$ (Since θ is angle between velocity and verticle)

4. (B) $eV_s = \dfrac{hc}{\lambda} - \phi = \dfrac{1240(nm)eV}{400(nm)} - 1.9\, eV = 1.2$ eV

$\Rightarrow \quad V_s = 1.2$ V

$\therefore$ The cesium ball can be charged to a maximum potential of 1.2 V.

5. (D) $\dfrac{1}{2}mv^2 = \dfrac{hc}{\lambda} - \phi$

$\dfrac{1}{2}mv'^2 = \dfrac{hc}{(3\lambda/4)} - \phi = \dfrac{4hc}{3\lambda} - \phi$

Clearly $v' > \sqrt{\dfrac{4}{3}}\, v$

6. Mass defect = (238.05079 – 234.04363 – 4.00260) u = 4.56×10^{-3} u
$= 4.56 \times 10^{-3} \times 1.66 \times 10^{-27} = 7.57 \times 10^{-30}$ kg
$mc^2 = 7.57 \times 10^{-30} \times 9 \times 10^{16} = 6.8 \times 10^{-13}$ J

7. $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$

$\therefore \quad E = \dfrac{h^2}{2m\lambda^2}$

$\Delta E = \dfrac{h^2}{2m}\left(\dfrac{1}{\lambda_1^2} - \dfrac{1}{\lambda_2^2}\right)$

Put $\lambda_1 = 0.5 \times 10^{-9}$ m
& $\lambda_2 = 2 \times 10^{-9}$ m and solve.

8. If electrons are accelereted through a potential difference V, the maximum energy of emitted photon could be

$E_{max} = eV. \quad \therefore \quad 10^5$ eV = eV

$\Rightarrow \quad V = 10^5$ V.

9. Energy of photon is given by mc^2 now the maximum energy of photon is equal to the maximum energy of electron = eV

hence, $mc^2 = ev \quad \Rightarrow \quad m = \frac{eV}{c^2} = \frac{1.6\times10^{-19}\times18\times10^3}{(3\times10^8)^2} = 3.2 \times 10^{-32}$ kg

10. The number of photons incident per unit time remains same hence saturation photo current remains same.

If frequency is doubled then kinetic energy of photo electrons is more than doubled.

11. $P_1 = P_2 = P$

$m_1v_1 = m_2v_2$

$$\left(\frac{P^2}{2m_1}\right)\Big/\left(\frac{P^2}{2m_2}\right) = \frac{64}{27}$$

$$\frac{m_2}{m_1} = \frac{64}{27} = \frac{v_1}{v_2}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{h/P_1}{h/P_2} = 1 : 1$$

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{27}{64}\right)^{1/3} = \frac{3}{4}$$

12. Type of particle emittion cannot be generalised for all reaction.

Hence α, β, γ particles may be emitted simultaneously.

13. The rate of accumulation of nuclei of X in the reactor can be given as

$$\frac{dN_X}{dt} = r - \lambda N_x$$

$$\Rightarrow \quad N_x = \frac{r}{\lambda}(1 - e^{-\lambda t})$$

Thus amount of N_x continuously increases with time hence brightness of bulb will continuously increase.

14. $\lambda = 2R = 2R_0A^{1/3}$

$$P = \frac{h}{\lambda} \quad \Rightarrow \quad E = \frac{P^2}{2m} = \frac{h^2}{2m(4R_0^2A^{2/3})}$$

$$E = \frac{(6.62\times10^{-34})^2}{2\times1.67\times10^{-27}\times4(1.3\times10^{-15})^2(128)^{2/3}} \text{ Joule}$$

= 4.72 MeV.

15. Activity after time t

$A = \lambda N_0 e^{-\lambda t}$

$A = \lambda A_i$

$A = \lambda$(initial activity)

$A \propto$ initial activity

16. Fraction of mass converted in energy

$$\frac{3\times 4.0025-12.0000}{3\times 4.0025}=\frac{0.0075}{12}=\frac{\text{Rate of loss of mass}}{\text{Rate of burning}}$$

Rate of burning $=\frac{12}{75\times 10^{-4}}$ Rate of loss of mass.

Rate of burning $=\frac{12}{75\times 10^{-4}}\times\frac{\text{Power output}}{C^2}$

$=\frac{12}{75\times 10^{-4}}\times\frac{4.5\times 10^{27}}{(3\times 10^{8})^2}$

$=\frac{12\times 4.5\times 10^{27}}{75\times 9\times 10^{12}}=\frac{54}{9\times 75}\times 10^{15}=\frac{2}{25}\times 10^{15}=8\times 10^{13}$ kg/s

17. Given, $\lambda = 0.173$

$T_{1/2}=\frac{\ln 2}{\lambda}=\frac{0.693}{0.173}\cong 4$

Also for $t=\frac{1}{0.173}$ year

Remaining nuclei $N = N_0e^{-1} = 0.37\ N_0$

Decay nuclei $= N_0 - N = 0.63\ N_0$.

18. Mass defect $\Delta m = 4m_H - m_{He} - 2m_e$

$Q = 0.027608\ u \times 932\frac{\text{MeV}}{u} = 25.7$ MeV

19. In ground state n = 1 and for first excited state n = 2

$KE=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r}(z=1)=\frac{14.4\times 10^{-10}}{2r}$ eV $\quad$ ($\because r = 0.53\ n^2$ Aº (z = 1))

$(KE)_1=\frac{14.4\times 10^{-10}}{2\times 0.53\times 10^{-10}}$ eV = 13.58 eV and $(KE)_2=\frac{14.4\times 10^{-10}}{2\times 0.53\times 10^{-10}\times 4}$ ev = 3.39 ev

$\therefore$ KE decreases by = 10.2 ev

$\therefore$ PE increases by = Excitation energy + Loss in kinetic energy = 10.2 + 10.2 = 20.4 ev

Now Angular momentum ; $L = mvr = \frac{nh}{2\pi}$

$\Rightarrow L_2 - L_1 = \frac{h}{2\pi}=\frac{6.6\times 10^{-34}}{6.28}=1.05\times 10^{-34}$ J–sec.

20. $\frac{\lambda_0}{\lambda_1}=4 \quad\Rightarrow\quad \frac{(Z_1-1)^2}{(Z-1)^2}=4 \quad\Rightarrow\quad Z_1 = 2Z-1$

$\frac{\lambda_0}{\lambda_2}=\frac{1}{4} \quad\Rightarrow\quad \frac{(Z_2-1)^2}{(Z-1)^2}=\frac{1}{4} \quad\Rightarrow\quad Z_2=\frac{Z+1}{2}$.

21. Time period $T_n = \frac{2\pi r_n}{V_n}$

$$T \propto \frac{n^2}{1/n}$$

i.e., $T \propto n^3$

$$T_{n_1} = 8T_{n_2}$$

$$n_1 = 2n_2$$

Hence, $n_1 = 2n_2$
Choice (b) and (c) are wrong.

22. $R = R_0A^{1/3}$
Radius of nucleus $R \propto A^{1/3}$
So, choice (b) is correct.

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{A \times 1.67 \times 10^{-27}}{\frac{4}{3}\pi R_0{}^3 A}$$

Density $\propto A°$
i.e., Density is indenpendent of mass number.
So, choices (a), (b) and (c) are correct and choice (d) is worng.

23. Pressure = $\frac{I}{C}$ (1 + r) where I is the Intensity

$F = \frac{P}{C}$ (1 + r) where P is the power

Impulse $I = \frac{E}{C}$ (1 + r)

where E is the Energy
r is the reflection coefficient.
and r = 1 for perfectly reflecting surface.
Choice (d) is wrong.

24. $I = \frac{1.06 z^2}{n^3}$ mA
For H atom z = 1 and first orbit n = 1
I = 1.06 mA. So, choice (a) is correct.

Magnetic field $B = \frac{12.5 z^3}{n^5}$ Tesla
B = 12.5 Tesla. So, choice (b) is correct.

$$\Delta E = 13.6\, z^2 \left[1 - \frac{1}{4}\right]$$

$= 13.6 \times \frac{3}{4} = 10.2$ eV. So, choice (c) is correct.

25. Energy incident in 1 m^2 in 1 sec.
E = 900 J

$\frac{hc}{\lambda}$ n × 1 × 3 × 10^8 = 900

n = 10^{13} photons/m^3
n = 10^4 photons/mm^3.

26. $\lambda_{min} = \frac{hc}{eV} = \frac{6.62\times10^{-34}\times3\times10^8}{1.6\times10^{-19}\times20\times10^3} = \frac{12375}{20\times10^{-3}}\ \text{Å} = 0.62\ \text{Å}$

27. $\sqrt{f}$ = a(Z–1)

$\sqrt{f}$ = a(31 – 1)

$\sqrt{\frac{25f}{x}}$ = a(51 –1)

x = 9

28. Q = [M (Ra^{226}) – M(Rn^{222}) – M(He^4)] × 931
= (226.025406 – 222.017574 – 4.002603) u × 931

= 0.005229 u × 931 $\frac{MeV}{u}$

Q = 4.87 MeV

29. Energy required to just remove the electron = 13.6 eV
∴ Energy required = 13.6 + 16.4 = 30 eV
If E be the photon energy 25%
E = 30 eV, E = 120eV = 24 × 5 eV.
X = 5 **Ans**

30. Using the law of radioactive decay, one can write $\frac{N_A(t)}{N_B(t)} = \frac{N_0\exp(-5\lambda t)}{N_0\exp(-\lambda t)} = \frac{1}{e}$.

Solving this one gets the results.

31. $r_n \ \alpha \ n^2$

$r_{n+2} = k(n+2)^2 \Rightarrow r_n = kn^2$

$r_{n-2} = k(n-2)^2$

$(n+2)^2 - n^2 = (n-2)^2 \Rightarrow$ n = 8

32. $\frac{hc}{\lambda}$ = 5 eV_0 + ϕ

$\frac{hc}{3\lambda}$ = eV_0 + ϕ $\Rightarrow$ $\frac{2hc}{3\lambda}$ = $4eV_0$

$\Rightarrow \phi = \frac{hc}{6\lambda}$

33. $E_n = -\dfrac{13.6\,eV}{n^2} = -1.51$ eV $\Rightarrow n = 3$

$\therefore L = 3\left(\dfrac{h}{2\pi}\right)$

34. $E = -13.6\,Z^2\left[\dfrac{1}{n_1^2}-\dfrac{1}{n_2^2}\right]$

$47.2 = -13.6\,Z^2\left[\dfrac{1}{2^2}-\dfrac{1}{3^2}\right]$

Z = 5

35. ${}^{229}_{90}X \rightarrow {}^{181}_{73}Y + a\,{}^{4}_{2}\alpha + b\,{}^{0}_{-1}\beta$

a =12

b = 7

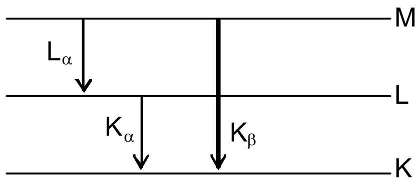
36. Let $\lambda_1 = \dfrac{\ln 2}{3}$ and $\lambda_2 = \dfrac{\ln 2}{4}$

$\dfrac{N/2}{N} = \dfrac{N_0 e^{-\lambda_1 t}}{N_0 e^{-\lambda_2 t}}$

t = 12

38 $\dfrac{1}{\lambda_{K\beta}} = \dfrac{1}{\lambda_{K\alpha}} + \dfrac{1}{\lambda_{L\alpha}}$

$\dfrac{1}{\lambda_{L\alpha}}$ = 5.6Å



eV = $\dfrac{hc}{\lambda}$

V = $\dfrac{hc}{e\lambda}$ = 31 × 10^3 volts

39. From Einstein photoelectic equation.

K = hν – φ

K' = hν – φ = n (hν - φ) + (n -1) φ

K' = nk + (n - 1)φ

From above expression K' > nK because φ can never be zero

40. Stopping potential is the measurment of maximum kinetic energy of emitted photoelectrons and kinetic energy of emitted photoelectrons is linearly with the frequency of incident light correponding (i,e corresponding to shortest wavelength, K.E is maximum).
Stopping potential is independent of intensity.

41. $\lambda_{min} = \frac{hc}{eV}$

$\lambda_{min} \propto \frac{1}{V}$

As λ_{min} decrease, V increases. So choice (a) is correct and the rest are wrong.

42. (A) Stopping potential in electron volts = $h\nu - \phi = 12 - 4 = 8$.

(B)) $\left(\frac{Z_1 - 1}{Z_2 - 1}\right)^2 = \frac{\lambda_2}{\lambda_1} = \left(\frac{85}{81}\right)^2$. Therefore $Z_1 = 86$ and $Z_2 = 82$

(C) Half life time of radioactive material is 4 min. For 80 gm to reduce to 20gm, two half life times are required.

(D) The binding energy per nucleon for helium in MeV is approximately $\frac{0.0302 \times 930}{4} \approx 7$

43. (P) Activity of the sample II becomes half in minimum time. Hence it has maximum disintegration constant.
(Q) Activity of the sample III takes maximum life to become half therefore it has maximum half - life.
(R)
(S) It can not be compared without information about atomic weight as energy radiated will depend upon no. of atoms, not upon amount of substance.

$A_0 = N_0\lambda_1 = N_0\lambda_2$

$\frac{A_0}{2} N_0 \lambda_3 \Rightarrow \lambda_1 = \lambda_2 = 2\lambda_3$

$N = \frac{N_0}{2^n} = \frac{N_0}{2^{\frac{t\lambda}{\mu 2}}}$

$\frac{N_3}{N_1} = 2^{\frac{t}{\mu 2}(\lambda_1 - \lambda_3)} > 1$

$N_3 > N_1$